# On the identification of creep processes at low stresses

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A number of experimental observations, supported by some theoretical considerations, have suggested the existence of a threshold stress for deformation, which signifies that an effective stress, rather than the applied stress, is responsible for the observed creep rate. In addition, recent experimental evidence shows that under some conditions of deformation the threshold stress varies strongly with temperature. The effect of the temperature dependence of the threshold stress on the measured values of the activation energy and the stress exponent is examined and it is shown that this temperature dependence may complicate various creep plots. Also, consideration of the nature of the interaction between different creep processes suggests that under certain experimental conditions it may be difficult to distinguish the difference between the operation of two sequential mechanisms and the operation of a threshold stress process.

## 1. Introduction

For simple materials, deformed at temperatures above about 0.5 $T_m$, where $T_m$ is the melting point of the material, the steady-state creep rate, $\dot{\epsilon}$, may, under a given set of experimental conditions, be represented by an equation of the form:

$$\dot{\epsilon} = B\frac{\sigma^n}{d^m}\exp\left(-\frac{Q_a}{RT}\right), \quad (1)$$

where $B$ is a constant, $\sigma$ is the applied stress, $n$ is the stress exponent, $d$ is the grain size, $m$ is the grain size sensitivity, $Q_a$ is the apparent activation energy, $R$ is the gas constant, and $T$ is the absolute temperature. In addition, recent analyses [1,2] of experimental data for a wide range of materials (metals and alloys) have shown that $B$ is not strictly independent of temperature and that it is preferable to represent the creep rate of a diffusion-controlled process by a dimensionless relationship of the form

$$\dot{\epsilon} = A\left(\frac{\mathbf{b}}{d}\right)^m \frac{DGb}{kT}\left(\frac{\sigma}{G}\right)^n \quad (2)$$

with

$$D = D_o \exp\left(-\frac{Q_D}{RT}\right), \quad (3)$$

where $A$ is a dimensionless constant, $\mathbf{b}$ is the Burgers vector, $D$ is the diffusion coefficient, $D_o$ is a frequency factor, $Q_D$ is the activation energy of the diffusion process that characterizes the creep behaviour, $G$ is the shear modulus and $k$ is Boltzmann's constant.

Rate controlling mechanisms of creep are generally identified by comparing the values of $Q$, $m$, and $n$, which are evaluated during creep experiments, with those values established for various basic processes. For example, according to Equation 1, a plot of the creep rate, $\dot{\epsilon}$, against the applied stress, $\sigma$, on a logarithmic scale for constant temperature and grain size yields a straight line with a slope representing the stress exponent, $n$. The value of $n$ can then be used to indicate whether a Newtonian process ($n = 1$) or a non-Newtonian process* ($n > 1$) governs the creep behaviour. However, this simple analysis may be complicated by the presence of a threshold stress for deformation, $\sigma_o$, which signifies that an effective stress, $\sigma_e (= \sigma - \sigma_o)$, rather than the applied stress, is responsible for the observed creep rate. An exam-

*Some dislocations models, when modified [3], predict $n = 1$, however, these mechanisms are open to several critisms.



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ple of such a complication was demonstrated recently [4] by considering low-stress creep data of large-grained aluminium. When those data were plotted as $\dot{\epsilon}$ against $\sigma$ on a logarithmic scale, the plot produced a straight line with a slope of 2, suggesting the presence of a deformation process having $n = 2$. However, when the data were replotted in terms of the effective stress, $\sigma_e$, since a threshold stress was measured, a modified Newtonian process ($\dot{\epsilon} \propto \sigma_e$) was correctly identified.

Not only may the failure to measure and incorporate the threshold stress into the creep analysis lead to erroneous constant values of the stress exponent, but it may also incorrectly imply the presence of two mechanisms operating sequentially. When two sequential creep processes contribute to the observed creep behaviour, a logarithmic plot of $\dot{\epsilon}$ against $\sigma$ over a narrow range of stresses shows a continuous increase in the value of the stress exponent as the applied stress decreases [5]. A similar increase is obtained if a threshold stress process exists [4]. In this paper, the difficulty in determining whether the creep behaviour at very low stresses arises from the operation of two sequential processes or from the operation of a threshold stress process is explored further using the concept of the activation energy for creep.

## 2. Analysis and discussion

The true activation energy, $Q_D$, is an important parameter in the deformation equation and must be determined in any attempt to identify the rate-controlling mechanism. According to Equation 2, $Q_D$ is defined as

$$Q_D = -R\left[\frac{\partial \ln D}{\partial T^{-1}}\right]_{d,\sigma}, \quad (4)$$

whereas the apparent activation energy, $Q_a$, is given by

$$Q_a = -R\left[\frac{\partial \ln \dot{\epsilon}}{\partial T^{-1}}\right]_{d,\sigma}. \quad (5)$$

The apparent activation energy, $Q_a$, is related to the true activation energy, $Q_d$, by the following expression [2]:

$$Q_D = Q_a + RT\left[1 + \left(\frac{n-1}{G}\right)\frac{\partial G}{\partial T}\right]. \quad (6)$$

Despite the frequent use of Equation 6 in correcting activation energies measured during creep, it appears that under most conditions, especially when $n < 5$, that $Q_D \simeq Q_a$ to within the accuracy of the experimental data. Accordingly, the correction term given by Equation 6 will be ignored and Equation 1, rather than Equation 2, will be used in the present analysis.

Under conditions of constant stress and grain size and under the operation of a single deformation process which can be represented by Equation 1, a plot of log $\dot{\epsilon}$ against $T^{-1}$ (Arrhenius plot) has a constant slope ($= -Q/2.3\,R$) which yields the value of the activation energy. More often, however, interaction between different creep processes may be significant over wide ranges of experimental conditions and as a result the Arrhenius plots of $\dot{\epsilon}$ against $T^{-1}$ may become complicated. Several forms of complications were recently discussed in detail [5–7], but there appears to be no evaluation of situations in which some of these complications may lead to ambiguities in the interpretation of the creep behaviour at low stresses. The situations for sequential processes and a threshold stress process are now considered for such an evaluation.

### 2.1. Sequential processes

When the operation of one deformation process is a prerequisite for the operation of the others, the deformation processes operate sequentially. For the situation where each process participates for a different time through any time period, $t$, and contributes an identical strain in order to keep the integrity of the material, the total creep rate is given by,

$$\frac{1}{\dot{\epsilon}_i} = \sum_i \frac{1}{\dot{\epsilon}_t}. \quad (7)$$

For two mechanisms operating sequentially, Equation 7 reduces to:

$$\dot{\epsilon}_t = \frac{\dot{\epsilon}_1 \dot{\epsilon}_2}{\dot{\epsilon} + \dot{\epsilon}_2} \quad (8)$$

It is clear that a plot of log $\dot{\epsilon}_t$ against $T^{-1}$ over a limited range of temperature would show a transition between the two modes of deformation.

For the purpose of illustration, and also comparison later with the behaviour of a threshold stress process, we shall consider two sequential processes, Type A and Type B, which proceed simultaneously with rates given by the equations:

$$\dot{\epsilon}_A = A\,\sigma \exp(-Q_A/RT) \quad (9)$$

and

$$\dot{\epsilon}_B = B\,\sigma^n \exp(-Q_B/RT). \quad (10)$$

The values of $A$, $Q_A$, $B$, $n$, and $Q_B$ have been chosen as $10^{-1}$, $83\,\text{kJ mol}^{-1}$, $5 \times 10^6$, 4, and $209\,\text{kJ mol}^{-1}$, respectively. For three different stress levels $\sigma_1$, $\sigma_2$, and $\sigma_3$ ($\sigma_3 > \sigma_2 > \sigma_1$), Fig. 1b shows the relationship between logarithm of $\dot{\epsilon}_t$ and $T^{-1}$ using Equation 8 along with Equations 9 and 10. In Fig. 1b the solid curve ABCD represents, the total creep rate*, and the broken lines represent the extrapolations of the two straight lines AB and CD arising from Processes A and B, respectively; the activation energies $Q_A$ and $Q_B$ are obtained from the slopes of these two lines, as indicated in Fig. 1. An examination of the plot shows that the activation energy changes from low ($Q_A$) to high ($Q_B$) value with decreasing temperature for constant $\sigma$, a characteristic which can be used to distinguish between sequential and independent summation; for independent summation ($\dot{\epsilon}_t = \Sigma\dot{\epsilon}_i$) the reverse is true.

## 2.2. A threshold stress process

A number of experimental findings, supported by some theoretical analyses, have suggested several sources which may account for the presence of a threshold stress, $\sigma_0$, during the Newtonian creep behaviour at low stresses ($\dot{\epsilon} = 0$ when $\sigma = \sigma_0$). These include: surface tension [8], oxidation effects [9,10], presence of particles at grain boundaries [11,12], and inefficiency of grain boundaries as vacancy sources and sinks [13]. Also, two recent theories of superplasticity [14,15] introduce a threshold stress into the constitutive equation of deformation to explain the experimentally-observed sigmoidal relationship between $\sigma$ and $\dot{\epsilon}$ at very low stresses. In the theory of Ashby and Verrall [14], the $\sigma_0$ term arises due to the work required to balance increases in grain-boundary area during the grain-switching event, whereas the theory of Gittus [15] attributes $\sigma_0$ to the interaction of interphase boundary superdislocations and grain-boundary ledges.

Theoretical analyses of the threshold stress processes associated with grain boundaries (presence of particles, inefficiency of grain boundaries, increase in grain-boundary area, etc...) predict a very weak temperature dependence, but recent experimental evidence, as reviewed by Burton [16], suggests that the threshold stress, at least during the Newtonian creep behaviour, varies strongly with temperature according to the following equation:

$$\sigma_0 = C \exp\left(\frac{Q_0}{RT}\right), \quad (11)$$

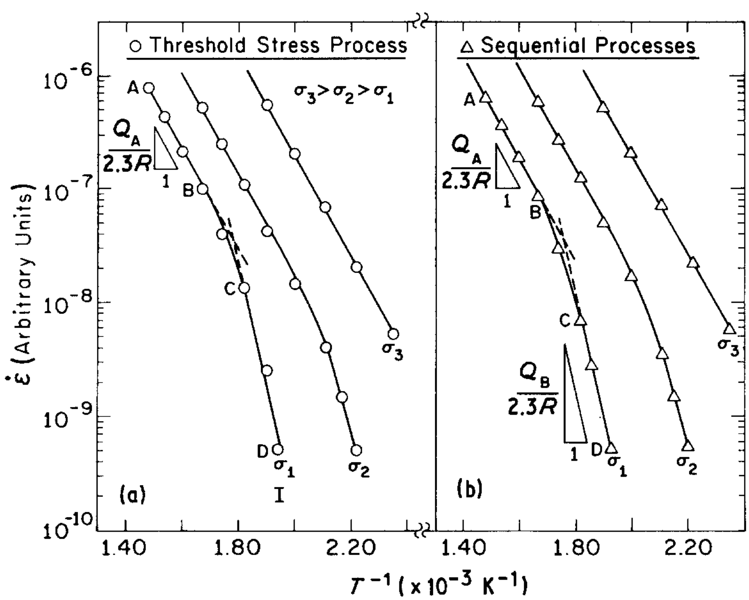


*Figure 1* Arrhenius plots [log $\dot{\epsilon}$ against $T^{-1}$ ($K^{-1}$)] at three constant stresses (a) for a threshold stress process and (b) for two sequential processes.

*This plot is experimentally obtained either by subjecting a specimen to a number of rapid changes in temperature, of the order of 10 K, while under a constant stress, or by using the logarithmic plots of $\dot{\epsilon}$ against $\sigma$ for several different temperatures.

where $C$ is a constant which depends on the grain size and shear modulus and $Q_0$ is an activation energy associated with $\sigma_0$. This strong temperature dependence of $\sigma_0$ would certainly complicate the Arrhenius plot of log $\dot{\epsilon}$ against $T^{-1}$, and it is interesting to examine whether this form of complication, under certain experimental conditions*, is similar to that produced by the sequential summation of two deformation processes.

Let us consider a hypothetical situation in which a modified Newtonian creep process, associated with a threshold stress process, controls the creep behaviour and obeys the following empirical law:

$$\dot{\epsilon}_t = C_t(\sigma - \sigma_0) \exp\left(\frac{-Q_A}{RT}\right) \qquad (12)$$

with

$$\sigma_0 = C_0 \exp\left(\frac{Q_0}{RT}\right). \qquad (13)$$

The values of $C_t, Q_A, C_0$ and $Q_0$ have been selected as $10^{-1}$, $83\,\text{kJ mol}^{-1}$, $10^{-2}$, and $33\,\text{kJ mol}^{-1}$, respectively, so that creep rates ascribable to the sequential summation of Equations 9 and 10, when plotted in Fig. 2 against the applied stress on a logarithmic scale, fall very close to those due to Equation 12 over the same experimental stress range; in Fig. 2, $\dot{\epsilon} = 5 \times 10^{-10}$ and $\sigma = 15$ (arbitrary units) are assumed to be the slowest detectable creep rate and the smallest applied stress, respectively. If it is taken that a creep rate can only be measured to within a limit of ±25%, this choice of $Q_A$, $Q_0$, $C_t$, and $C_0$ shows that the two sets of points (circles and triangles) are experimentally indistinguishable for four different temperatures.

In Fig. 2a, the logarithm of creep rate arising from the modified creep process, given by Equation 12, is plotted against $T^{-1}$ for three different stress levels $\sigma_1$, $\sigma_2$ and $\sigma_3$ ($\sigma_3 > \sigma_2 > \sigma_1$), which are comparable to those used in plotting Fig. 1b. The plot reveals three important points. First, for high strain rates (high stresses), the activation energy for creep is equal to $Q_A$ and $\sigma_0$ has no significant effect on its value. Second, for a constant value of $\sigma$, as the creep rate decreases with an increase in $T^{-1}$, the slope of the Arrhenius plot increases, suggesting an increase of $Q$ with decreasing temperature; straightforward treatment

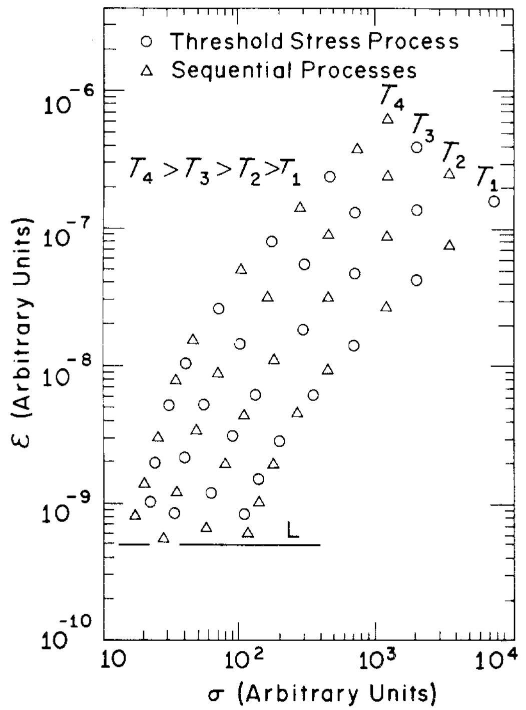


*Figure 2* Log (strain rate) against log (applied stress) at four constant temperatures for a threshold stress process and for two sequential processes.

shows that the apparent activation energy, $Q$, determined from the plot, is related to $Q_A$ and $Q_0$ by the following expression:

$$Q = Q_A + Q_0 \bigg/ \left(\frac{\sigma}{\sigma_0} - 1\right)$$

$$(\sigma \gg \sigma_0, Q \simeq Q_A\,; \text{and } \sigma = \sigma_0, Q \to \infty). \qquad (14)$$

Third, if measurements of the activation energy were only feasible over a narrow range of temperature and if $\dot{\epsilon} = 5 \times 10^{-10}$ (arbitrary units) represents the slowest detectable creep rate, the computed Curve I, for example, would, in view of normal experimental scatter, be indistinguishable from the fit using two straight lines, AB and CD, and a transition knee, BC. This fit, despite its invalidity for $\dot{\epsilon} < 5 \times 10^{-10}$, yields a plot that stimulates the sequential situation of Fig. 1b, as discussed earlier.

While a hypothetical modified Newtonian process, which incorporates a threshold stress, is used to illustrate the present argument, the same trend shown in Figs 1 and 2 would also be obtained

*These experimental conditions may include: a lower limit for the external load which can be applied to a specimen due to the significance of friction at low stresses, a narrow range of temperature which is dictated by the stability of the material being tested, or lack of very accurate measuring devices to monitor extremely low creep rates.

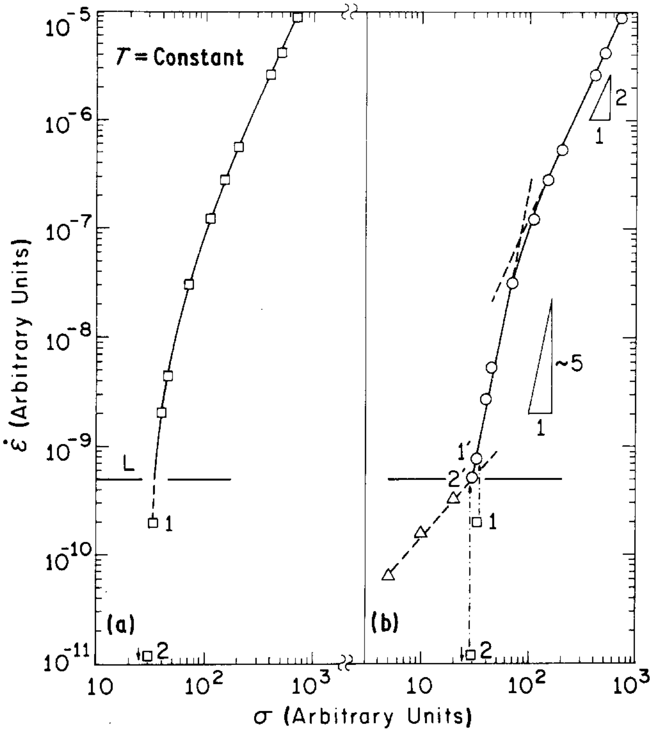


*Figure 3* Log (strain rate) against log (applied stress) (a) for a power-law threshold stress process and (b) for independently summed Newtonian and power-law threshold stress processes.

if a modified power-law process, $\dot{\epsilon} = A(\sigma - \sigma_0)^n$, was assumed to be operative. This situation is not unrealistic since experimental measurements made during superplastic flow of a duplex stainless steel [17] have revealed the presence of a threshold stress that decreased with increasing temperature. Fig. 3a represents a logarithmic plot of $\dot{\epsilon}$ against $\sigma$ for the following power-law process:

$$\dot{\epsilon} = C_{t1} (\sigma - \sigma_{01})^2 \exp\left(\frac{-Q_{A1}}{RT}\right) \quad (15)$$

with

$$\sigma_{01} = C_{01} \exp\left(\frac{Q_{01}}{RT}\right). \quad (16)$$

The values of $C_{t1}$, $Q_{A1}$, $C_{01}$, and $Q_{01}$ have been selected as $9.4 \times 10^{-3}$, $84\,\text{kJ}\,\text{mol}^{-1}$, $10^{-2}$, and $33\,\text{kJ}\,\text{mol}^{-1}$, respectively. It is apparent, from Fig. 3a, that if sufficient experimental precision was available at very low creep rates, the data of the modified power-law process, unlike those of the sequential summation, would exhibit a rapid and continuous increase in the stress exponent with decreasing $\sigma$ at constant temperature; Point 2 is marked by a downward arrow since it is out of scale. However, this trend may be masked, and the identification process may become difficult, if an independent mechanism of deformation intervenes at very low stresses. This point is illustrated in Section 2.3.

## 2.3 The effect of an independent low-stress process

Let us assume that creep rates arise from the independent summation ($\dot{\epsilon}_t = \dot{\epsilon}_1 + \dot{\epsilon}_2$) of Equation 15 and the following rate equation:

$$\dot{\epsilon} = C_{t2} \left[\sigma - C_{02} \exp\left(\frac{Q_{02}}{RT}\right)\right] \exp\left(\frac{Q_{A2}}{RT}\right). \quad (17)$$

The values of $C_{t2}$, $C_{02}$, $Q_{02}$, and $Q_{A2}$ have been chosen as $8.7 \times 10^{-3}$, $3.5 \times 10^{-3}$, $25\,\text{kJ}\,\text{mol}^{-1}$, and $84\,\text{kJ}\,\text{mol}^{-1}$, respectively. In Fig. 3, the independent summation is represented by circles, whereas the contributions from Equations 15 and 17 are shown by squares and triangles, respectively. Three observations are noted in Fig. 3b. First, the creep rates at high stresses are approximately given by Equation 15, since the contribution from the low-stress process is negligible. Second, the two lowest points (Points 1 and 2), ascribable to the operation of the threshold stress process, are displaced to Points 1′ and 2′, owing to the significant contribution of Equation 17. Third, the trend with the new positions of Points 1 and 2 can now be

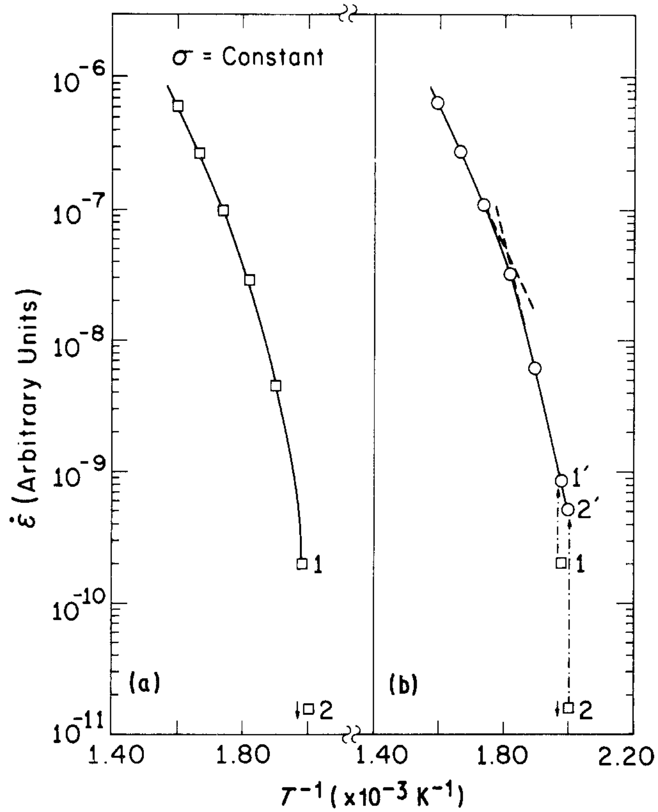


*Figure 4* Arrhenius plots [(log $\dot{\epsilon}$ against $T^{-1}$ ($K^{-1}$)] at constant stress (a) for a power-law threshold stress process and (b) for independently summed Newtonian and power-law threshold stress processes.

approximated by two straight lines, having slopes of 2 and 5, connected by a transition curve. This approximation incorrectly implies the operation of two sequential processes. Moreover, the situation becomes even more misleading if a limiting creep rate, represented by a horizontal line (labelled L) in Fig. 3, exists, or if, for some reason, investigation of the creep behaviour of the material is not pursued at lower stresses.

The operation of a second, independent mechanism at low stresses would also cause an error in the interpretation of the results of activation energy measurements, as demonstrated in Fig. 4. Fig. 4a represents log $\dot{\epsilon}$ against $T^{-1}$ for the threshold stress process given by Equation 15 and shows that the data exhibit a rapid and continuous increase in the value of the activation energy with decreasing temperature at constant stress. In Fig. 4b the independent summation of the threshold stress process and the mechanism represented by Equation 17 is shown, and it is evident that the significant contribution of this independent mechanism at low stresses causes the displacement of the two points, Points 1 and 2, to higher creep rates, Points 1′ and 2′. Again, a straight-line fit, which is possible due to the new positions of Points 1 and 2, would erroneously lead to a situation almost identical to that produced by the sequential summation (two straight lines connected by a transition knee).

It is clear from the above discussion that under certain experimental conditions the similarity between the behaviour of two sequential processes and that of a threshold stress process, which is sensitive to temperature, is so strong that creep characteristics other than the activation energy and the stress exponent must be sought if the creep behaviour of the material is to be identified. Among these characteristics, substructural analyses should be invaluable in providing guiding information.

## 3. Conclusion

It is demonstrated that under certain experimental conditions the values of the activation energy inferred from the Arrhenius plots of log $\dot{\epsilon}$ against $T^{-1}$ do not provide a sufficiently good criterion to distinguish between the operation of two sequential processes and the operation of a threshold stress process that depends strongly on temperature. Also, the identification of such a threshold stress process becomes even more difficult if the creep behaviour of the material includes contributions from other independent mechanisms.


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